

## Further development of the preceding Gaussian-pencil-beam-model used for calculation of the in-water dose caused by clinical electron-beam irradiation

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### Abstract

**Purposes:** We perform further development for our previous Gaussian-pencil-beam-model used for calculating the electron dose in water under clinical electron-beam irradiation. The main purpose is to evaluate accurately the parallel beam depth-doses at deep depths beyond about the extrapolated range ( $R_p$ ) under an infinite field. **Methods:** Sets of parallel beam depth-doses under an infinite field were reconstructed for beams of  $E=6, 12,$  and  $18$  MeV in light of the electron Monte Carlo (eMC) datasets reported by Wieslander and Knöös (2006), separating the datasets into the direct electron beam and direct-plus-indirect electron beam groups. The datasets at the deep depths were then reconstructed using each factor of  $R_{scale}^{OAD}$ . **Results and conclusions:** The following results were obtained by comparing the calculated datasets of depth dose (DD) and off-axis dose (OAD) with the eMC datasets: (i) The further revised Gaussian pencil beam model is of practical use without using complicated correction factors; and (ii) The DD and OAD datasets are yielded effectively over wide ranges of depths and off-axis distances.

**Keywords:** Gaussian pencil beam model; dose calculation; electron MC; electron beams; linear accelerator

### Research highlights

The dose in water caused by the clinical electron-beam irradiation is mainly composed of the doses due to the direct electrons, the indirect electrons, and the contaminant X-rays. In light of the electron Monte Carlo (eMC) datasets reported by Wieslander and Knöös (2006), this paper describes further development of the preceding Gaussian-pencil-beam-model for calculating doses in water. The main subject is how to evaluate accurately the parallel beam depth-doses at deep depths beyond about the extrapolated range ( $R_p$ ) under an infinite field. Characteristic shapes were yielded effectively at shallow and deep depths for both of the depth dose curves and the off-axis dose profiles.

### Introduction

In 2022, we reported [1] a revised Gaussian-pencil-beam model, which was constructed on the basis of electron Monte Carlo (eMC) datasets performed by Wieslander and Knöös [2, 3] for 6-, 12-, and 18-MeV electron beams taking  $10 \times 10$  cm<sup>2</sup> and  $10 \times 10/14 \times 14$  cm<sup>2</sup> applications. However, we realize that there remain two problems: (a) Parallel-beam depth-dose datasets ( $D_{\infty}$ ) at deep depths in an infinitely broad field; and (b) Characteristic shape differences of the off-axis dose (OAD) profiles at shallow and deep depths.

For resolving these problems, we use the almost same dose calculation procedures as in the previous paper, only excepting the usage purpose of the  $R_{scale}^{OAD}$  factor [1] (this factor is originally introduced for recalculating reasonable OAD datasets at deep depths from ones illustrated in paper diagrams). Although this paper also utilizes a parallel-beam depth-dose dataset ( $D_{\infty}$ ) of infinite field for a given irradiation, the parallel-beam depth-dose dataset ( $D_{\infty}$ ) is partially recalculated using the  $R_{scale}^{OAD}$  factor for a point that is situated beyond about the extrapolated range

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( $R_p$ ). Actually, reporting this technique is the main subject of the paper.

**Materials and methods**

Based on the depth dose (DD) and off-axis dose (OAD) datasets of the W-K eMC work [3], we similarly develop this study. Here, we would like to emphasize that each of the DD or OAD datasets is normalized with a dose of 1.0 Gy per 100 MU at the maximum dose depth ( $d_{max}$ ) caused under the use of an open electron applicator of  $A_{appl} = 20 \times 20 \text{ cm}^2$ . This paper is also use the same dose unit of Gy/100 MU for each of the DD and OAD datasets.

We use  $KK$  numbers of  $KK=1$  to 24 for the DD and OAD datasets, originally introduced in the previous paper on compiling and handling of the beam energy ( $E$ ), the electron applicator ( $A_{appl}$ ), and the utilized electron Monte Carlo (eMC). It should be noted that the eMC-based dose is separated into the dose due to (i) the direct electrons getting no interactions with the electron applicator, the dose due to (ii) the indirect electrons getting interactions with the electron applicator, and the dose due to (iii) the contaminant X-rays from within the treatment head. This paper is described only taking the direct electron beams and the direct-plus-indirect electron beams (Tables 1 and 2). The Supplementary Figure (Supp. Fig.) numbers below are the corresponding ones described in the paper of the W-K eMC work using standard and commercial eMC techniques (it should be noted that the OAD datasets for a given irradiation are yielded on two horizontal planes at shallow and deep  $Z_c$  depths).

For the direct electron beams (Table 1), the standard eMC datasets are classified into:

Under Supp. Fig. 3(a)-DD, we set  $KK=1$  for Supp. Fig. 3(b)-OAD ( $Z_c=1 \text{ cm}$ ) and  $KK=2$  for Supp. Fig. 3(c)-OAD ( $Z_c=5 \text{ cm}$ );

Under Supp. Fig. 5(a)-DD, we set  $KK=3$  for Supp. Fig. 5(b)-OAD ( $Z_c=2 \text{ cm}$ ) and  $KK=4$  for Supp. Fig. 5(c)-OAD ( $Z_c=10 \text{ cm}$ );

Under Supp. Fig. 3(d)-DD, we set  $KK=5$  for Supp. Fig. 3(e)-OAD ( $Z_c=3 \text{ cm}$ ) and  $KK=6$  for Supp. Fig. 3(f)-OAD ( $Z_c=15 \text{ cm}$ ).

Similarly, the commercial eMC datasets are classified into:

Under Supp. Fig. 3(a)-DD, we set  $KK=7$  for Supp. Fig. 3(b)-OAD ( $Z_c=1 \text{ cm}$ ) and  $KK=8$  for Supp. Fig. 3(c)-OAD ( $Z_c=5 \text{ cm}$ );

Under Supp. Fig. 5(a)-DD, we set  $KK=9$  for Supp. Fig. 5(b)-OAD ( $Z_c=2 \text{ cm}$ ) and  $KK=10$  for Supp. Fig. 5(c)-OAD ( $Z_c=10$ );

Under Supp. Fig. 3(d)-DD, we set  $KK=11$  for Supp. Fig. 3(e)-OAD ( $Z_c=3 \text{ cm}$ ) and  $KK=12$  for Supp. Fig. 3(f)-OAD ( $Z_c=15 \text{ cm}$ ).

For the direct-plus-indirect electron beams (Table 2), the standard eMC datasets are classified into:

Under Supp. Fig. 3(a)-DD, we set  $KK=13$  for Supp. Fig. 3(b)-OAD ( $Z_c=1 \text{ cm}$ ) and  $KK=14$  for Supp. Fig. 3(c)-OAD ( $Z_c=5 \text{ cm}$ );

Under Supp. Fig. 5(a)-DD, we set  $KK=15$  for Supp. Fig. 5(b)-OAD ( $Z_c=2 \text{ cm}$ ) and  $KK=16$  for Supp. Fig. 5(c)-OAD ( $Z_c=10 \text{ cm}$ );

Under Supp. Fig. 3(d)-DD, we set  $KK=17$  for Supp. Fig. 3(e)-OAD ( $Z_c=3 \text{ cm}$ ) and  $KK=18$  for Supp. Fig. 3(f)-OAD ( $Z_c=15 \text{ cm}$ ).

**Table 1** Values of the constants used in equation 1 for the direct electron beams (the  $KK$  numbers are the same as in the previous paper [1]).

For direct electron beams	$D_a$ (Gy/100 MU)	$Z_a$ (cm)	$D_b$ (Gy/100 MU)	$Z_b$ (cm)	$R_{scale}^{OAD}(Z_c)$ (no unit)
(i) Standard eMC					
$KK=1$ & 2 ( $E=6 \text{ MeV}$ )	1.45E-03	5	8.36E-02	3.35	0.619
$KK=3$ & 4 ( $E=12 \text{ MeV}$ )	3.38E-03	10	1.08E-02	6.99	39.063
$KK=5$ & 6 ( $E=18 \text{ MeV}$ )	5.38E-03	15	1.30E-02	9.72	36.115
(ii) Commercial eMC					
$KK=7$ & 8 ( $E=6 \text{ MeV}$ )	1.49E-03	5	8.88E-02	3.33	1.548
$KK=9$ & 10 ( $E=12 \text{ MeV}$ )	3.00E-03	10	1.08E-02	7.00	34.278
$KK=11$ & 12 ( $E=18 \text{ MeV}$ )	5.63E-03	15	1.41E-02	9.89	50.819

Similarly, the commercial eMC datasets are classified into:

Under Supp. Fig. 3(a)-DD, we set  $KK=19$  for Supp. Fig. 3(b)-OAD ( $Z_c=1 \text{ cm}$ ) and  $KK=20$  for Supp. Fig. 3(c)-OAD ( $Z_c=5 \text{ cm}$ );

Under Supp. Fig. 5(a)-DD, we set  $KK=21$  for Supp. Fig. 5(b)-OAD ( $Z_c=2 \text{ cm}$ ) and  $KK=22$  for Supp. Fig. 5(c)-OAD ( $Z_c=10 \text{ cm}$ );

Under Supp. Fig. 3(d)-DD, we set  $KK=23$  for Supp. Fig. 3(e)-OAD ( $Z_c=3 \text{ cm}$ ) and  $KK=24$  for Supp. Fig. 3(f)-OAD ( $Z_c=15 \text{ cm}$ ).

**Table 2** Values of the constants used in equation 1 for the direct & indirect electron beams (the  $KK$  numbers are the same as in the previous paper [1]).

For direct & indirect electron beams	$D_a$ (Gy/100 MU)	$Z_a$ (cm)	$D_b$ (Gy/100 MU)	$Z_b$ (cm)	$R_{scale}^{OAD}(Z_c)$ (no unit)
(i) Standard eMC					
$KK=13$ & 14 ( $E=6 \text{ MeV}$ )	1.33E-03	5	8.96E-02	3.36	0.159
$KK=15$ & 16 ( $E=12 \text{ MeV}$ )	3.65E-03	10	3.01E-02	6.94	0.214
$KK=17$ & 18 ( $E=18 \text{ MeV}$ )	5.81E-03	15	1.44E-02	9.91	1.21E+02
(ii) Commercial eMC					
$KK=19$ & 20 ( $E=6 \text{ MeV}$ )	1.33E-03	5	8.96E-02	3.36	0.156
$KK=21$ & 22 ( $E=12 \text{ MeV}$ )	3.25E-03	10	3.77E-02	6.36	0.180
$KK=23$ & 24 ( $E=18 \text{ MeV}$ )	5.66E-03	15	1.23E-02	9.88	1.72E+02

We propose another usage of the  $R_{scale}^{OAD}$  factor, which is used for obtaining datasets of parallel beam depth-dose ( $D_\infty$ ) in an infinitely broad field using a mathematical expression as a function of depth  $Z$  in a region beyond about the extrapolated range ( $R_p$ ).

For constructing such datasets of parallel beam depth-dose ( $D_\infty$ ), we use again the same DD datasets plotted in graphs of the paper reported by Wieslander and Knöös [3] (the construction method is the same as reported in the previous paper [1]). It has been found that, as shown later

in Supp. Fig. 1, the reconstructed  $D_{\infty}$  datasets reported in the previous paper usually contain relatively large errors in the low-dose region at deep depths beyond about the extrapolated range ( $R_p$ ). In order to avoid this situation, the previous paper performed the dose calculation introducing the  $R_{scale}^{OAD}$  factor as a dose correction factor only at each specific depth.

This paper proposes a method to reconstruct a more reasonable  $D_{\infty}$  dataset at deep depths for each beam irradiation. Supp. Fig. 1 shows its procedure (utilizing the DD dataset of  $KK=15$  &  $16$ , as illustrated below in Supp. Fig. 5b). The orange line shows a raw dataset as used in the previous paper, and the blue line shows its modified dataset proposed in the present dose calculation, setting two points along the beam axis. Let the point at  $Z=Z_a$  indicate the position of the dose ( $D_a$ ) that is equal to the dose obtained using the factor of  $R_{scale}^{OAD}$  as proposed in the previous paper, and let the point at  $Z=Z_b$  ( $<Z_a$ ) indicate the common position of another reasonable dose of  $D_b$  estimated by eye on both dose lines. Then along the blue line for  $Z \geq Z_b$ , we set

$$D_{\infty}(Z) = D_b \exp[-\alpha(Z-Z_b)], \quad (\text{Eq.1})$$

where  $\alpha$  is a constant, determined by the doses of  $D_a$  and  $D_b$ , as pointed up by using yellow dots in Supp. Fig. 1, at the corresponding two positions of  $Z_a$  and  $Z_b$ .

Table 1 lists sets of ( $D_a$ ,  $Z_a$ ), ( $D_b$ ,  $Z_b$ ), and  $R_{scale}^{OAD}(Z_a)$  values for  $KK=1-12$  based on the standard or commercial eMC for the direct electron beams of  $E=6, 12$ , and  $18$  MeV. Similarly, Table 2 lists the corresponding datasets for  $KK=13-24$  taking the direct and indirect electron beams. It should be noted that the  $Z_a$  point is situated beyond the extrapolated range ( $R_p$ ) for each irradiation [1], and that the  $R_{scale}^{OAD}$  factor at each  $Z_a$  depth all takes much smaller or larger values than unity (we need no correction for  $D_a$  in case of  $R_{scale}^{OAD}=1$ ).

We use the same dose calculation procedure as the previous one [1], only except the purpose of the usage of the  $R_{scale}^{OAD}$  factor as described above.

Supp. Fig. 2 shows how the effective square field of  $A_c^{eff} = S_c^{eff} \times S_c^{eff}$  at each dose calculation  $Z_c$  depth (as shaped using a square electron applicator ( $A_{appl}$ )) is divided equally into small sections of  $40 \times 40$ . In the previous paper [1], we has reported that the  $S_c^{eff}$  function spreads exponentially with  $Z_c$  depth in water. Almost the same phenomenon is illustrated in a Monte-Carlo drawing picture on the cover of Klevenhagen's textbook [4]. However, the perfect square-shaped fields would not be held with increasing depth in phantom like the case of the primary X-ray beam intensity distribution.

## Results and discussion

First, we describe how the parallel-beam depth-dose datasets ( $D_{\infty}$ ) of infinite field are varied in (i) the whole  $Z$  region and (ii) the deep  $Z$  region on large scale, as follows:

Supp. Fig. 3a-c shows the *direct* electron beam cases on the *standard* eMC for (a)  $KK=1$  &  $2$  ( $E=6$  MeV), (b)  $KK=3$  &  $4$  ( $E=12$

MeV), and (c)  $KK=5$  &  $6$  ( $E=18$  MeV). Similarly, Supp. Fig. 4a-c shows the cases on the *commercial* eMC for (a)  $KK=7$  &  $8$  ( $E=6$  MeV), (b)  $KK=9$  &  $10$  ( $E=12$  MeV), and (c)  $KK=11$  &  $12$  ( $E=18$  MeV).

Supp. Fig. 5a-c shows the *direct & indirect* electron beam cases on the *standard* eMC for (a)  $KK=13$  &  $14$  ( $E=6$  MeV), (b)  $KK=15$  &  $16$  ( $E=12$  MeV), and (c)  $KK=17$  &  $18$  ( $E=18$  MeV). Similarly, Supp. Fig. 6a-c shows the cases on the *commercial* eMC for (a)  $KK=19$  &  $20$  ( $E=6$  MeV), (b)  $KK=21$  &  $22$  ( $E=12$  MeV), and (c)  $KK=23$  &  $24$  ( $E=18$  MeV).

It could be understood more clearly from the diagrams on large scale that each blue line at depths of  $Z \geq Z_b$  is connected smoothly with the corresponding orange line at depths of  $Z < Z_b$ .

Lastly, we describe how the OAD curve pattern varies with  $Z_c$  depth by setting regions of relatively (i) shallow and (ii) deep  $Z_c$  depths, as follows:

Supp. Fig. 7a-c shows the *direct* electron beam cases on the *standard* eMC for (a)  $KK=1$  &  $2$  ( $E=6$  MeV), (b)  $KK=3$  &  $4$  ( $E=12$  MeV), and (c)  $KK=5$  &  $6$  ( $E=18$  MeV). Similarly, Supp. Fig. 8a-c shows the cases on the *commercial* eMC for (a)  $KK=7$  &  $8$  ( $E=6$  MeV), (b)  $KK=9$  &  $10$  ( $E=12$  MeV), and (c)  $KK=11$  &  $12$  ( $E=18$  MeV).

Supp. Fig. 9a-c shows the *direct & indirect* electron beam cases on the *standard* eMC for (a)  $KK=13$  &  $14$  ( $E=6$  MeV), (b)  $KK=15$  &  $16$  ( $E=12$  MeV), and (c)  $KK=17$  &  $18$  ( $E=18$  MeV). Similarly, Supp. Fig. 10a-c shows the cases on the *commercial* eMC for (a)  $KK=19$  &  $20$  ( $E=6$  MeV), (b)  $KK=21$  &  $22$  ( $E=12$  MeV), and (b)  $KK=23$  &  $24$  ( $E=18$  MeV).

It can be seen from the OAD curves that the shapes in the very shallow  $Z_c$  depths (less than  $\sim 1E-04$  cm) form triangles, the ones in the middle  $Z_c$  depths form squarish trapezoids, and the ones in the deep  $Z_c$  depths form round trapezoids.

The revised Gaussian-pencil-beam-model uses a mathematical  $\sigma_r$  expression, being reconstructed based on datasets of  $\sigma_z^p$  for  $E = 6, 10, 14$ , and  $20$  MeV as reported by Bruinvis et al. [5]. On the other hand, judging from the OAD curves at very shallow depths for each of the  $6, 12$ , and  $18$  MeV beam energies, we should see steep DD descents toward each zero depth. Actually, we have obtained a fact that each calculated DD curve forms zero dose values at very shallow  $Z_c$  depths (less than  $\sim 1E-15$  cm), where it should be noted that we do not use any set of parallel beam depth-doses of infinite field that descends very sharply near the zero depth, as seen from Supp. Fig. 3-6. Examining Khan's text book [6], such dose descents are not described (details will be reported in the next article). We would like to let it be a great subject when taking inhomogeneous phantoms for dose calculations with this Gaussian-pencil-beam-model.

## Conclusions

We conducted two supplementary studies for the previous paper: one is for the parallel-beam depth-dose dose ( $D_{\infty}$ ) at deep depths in an infinitely broad field, and the other is for the characteristic shape differences of off-axis dose

(OAD) profiles at shallow and deep depths, including in the very shallow depths (less than  $\sim 1 \text{E-}04$  cm). We believe that these two studies must be important for dose calculations especially when using heterogeneous phantoms or when considering the dose calculation on a cell-by-cell basis.

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### Conflicts of interest

This study was carried out in collaboration with Technology of Radiotherapy Corporation, Tokyo, Japan. This sponsor had no control over the interpretation, writing, or publication of this work.

### Supplementary data

Supplementary material associated with this article can be found at <http://dx.doi.org/10.14312/2399-8172.2023-1>.

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