

A revised Gaussian pencil beam model for calculation of the in-water dose caused by clinical electron-beam irradiation

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Appendix A

Comments on the σ_r and σ_z^p functions

For a pencil beam irradiation to a water phantom, we take X and Y rectangular coordinate axes on a plane at a phantom depth Z along the beam axis, where the X and Y origins are taken at the same point on the beam axis. For pencil beam algorithms based on the multiple scattering theory, the two sigma functions of $\sigma_r^2(Z)$ and $\sigma^2(Z)$ are introduced, where $\sigma_r^2(Z)$ is the mean square radial displacement of electrons as a result of multiple Coulomb scattering on the Z -plane, and $\sigma^2(Z)$ is the mean square lateral displacement ($\overline{\Delta x^2}$ or $\overline{\Delta y^2}$) projected on to the X -axis or on to the Y -axis, respectively, at depth Z [1, 8]. Letting $\overline{\Delta r^2} = \overline{\Delta x^2} + \overline{\Delta y^2}$ and $\overline{\Delta x^2} = \overline{\Delta y^2}$, we obtain $\sigma_r^2(Z) = 2\sigma_x^2(Z) = 2\sigma_y^2(Z) [= 2\sigma^2(Z)]$. Accordingly, equation (10) can be rewritten as

$$\begin{aligned} \Delta D(X_c, Y_c, Z_c) &= D_{\text{para}}(0, 0, Z_c^{\text{fan}} : A_c = \infty) \\ &\times \frac{1}{4} \left(\operatorname{erf} \frac{\Delta X_c'/2 + X_c}{\sqrt{2}\sigma(Z_c^{\text{fan}})} + \operatorname{erf} \frac{\Delta X_c'/2 - X_c}{\sqrt{2}\sigma(Z_c^{\text{fan}})} \right) \left(\operatorname{erf} \frac{\Delta Y_c'/2 + Y_c}{\sqrt{2}\sigma(Z_c^{\text{fan}})} \right. \\ &\left. + \operatorname{erf} \frac{\Delta Y_c'/2 - Y_c}{\sqrt{2}\sigma(Z_c^{\text{fan}})} \right). \end{aligned} \quad (\text{Eq. A1})$$

By examining Appendix 3 (Determination of treatment beam parameter σ_z^p) in the paper by Bruinvis et al. [7], we can tell the equality of $\sigma_r(Z) = \sigma_z^p$.