Original research

A revised Gaussian pencil beam model for calculation of the in-water dose caused by clinical electron-beam irradiation

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Appendix A

Comments on the σ_{r} and σ_z^{p} functions

For a pencil beam irradiation to a water phantom, we take *X* and *Y* rectangular coordinate axes on a plane at a phantom depth Z along the beam axis, where the *X* and *Y* origins are taken at the same point on the beam axis. For pencil beam algorithms based on the multiple scattering theory, the two sigma functions of $\sigma_{\mathrm{r}}^2(Z)$ and $\sigma^2(Z)$ are introduced, where $\sigma_{\mathrm{r}}^2(Z)$ is the mean square radial displacement of electrons as a result of multiple Coulomb scattering on the *Z*-plane, and $\sigma^2(Z)$ is the mean square lateral displacement ($\overline{\Delta x^2}$ or $\overline{\Delta y^2}$) projected on to the *X*-axis or on to the *Y*-axis, respectively, at depth *Z* [1, 8]. Letting $\overline{\Delta r^2} = \overline{\Delta x^2} + \overline{\Delta y^2}$ and $\overline{\Delta x^2}$=$\overline{\Delta y^2}$, we obtain $\sigma_{\mathrm{r}}^2(Z) = 2\sigma_x^2(Z) = 2\sigma_y^2(Z)$ [$= 2\sigma^2(Z)$]. Accordingly, equation (10) can be rewritten as

$$\Delta D(X_{\mathrm{c}}, Y_{\mathrm{c}}, Z_{\mathrm{c}}) = D_{\mathrm{para}}\left(0,0, Z_{\mathrm{c}}^{\mathrm{fan}} : A_{\mathrm{c}} = \infty\right)$$
$$\times \frac{1}{4}\left(\operatorname{erf} \frac{\frac{\Delta X_{\mathrm{c}}'}{2} + (X_{\mathrm{c}} - X_{\mathrm{c}}')}{\sqrt{2}\sigma(Z_{\mathrm{c}}^{\mathrm{fan}})} + \operatorname{erf} \frac{\frac{\Delta X_{\mathrm{c}}'}{2} - (X_{\mathrm{c}} - X_{\mathrm{c}}')}{\sqrt{2}\sigma(Z_{\mathrm{c}}^{\mathrm{fan}})}\right)$$
$$\left(\operatorname{erf} \frac{\Delta Y_{\mathrm{c}}'/2 + (Y_{\mathrm{c}} - Y_{\mathrm{c}}')}{\sqrt{2}\sigma(Z_{\mathrm{c}}^{\mathrm{fan}})} + \operatorname{erf} \frac{\Delta Y_{\mathrm{c}}'/2 - (Y_{\mathrm{c}} - Y_{\mathrm{c}}')}{\sqrt{2}\sigma(Z_{\mathrm{c}}^{\mathrm{fan}})}\right) \qquad (\mathrm{Eq.\,A1})$$

By exanimating Appendix 3 (Determination of treatment beam parameter σ_z^{p}) in the paper by Bruinvis et al. [7], we can tell the equality of $\sigma_{\mathrm{r}}(Z)$=σ_z^{p}.